Official URL
https://doi.org/10.1109/MILCOM.2017.8170743

New Perspectives for Coded Continuous Phase Modulations for NarrowBand Waveforms: Iterative Versus Non-Iterative Solutions

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***Abstract*—NarrowBand Waveforms (NBWF) are often used in VHF or UHF tactical communications. For these kinds of waveforms, low latency and robust data rates result in short codeword lengths that are challenging in terms of channel coding. Usually, serially concatenated convolutional code and continuous phase modulation (CC-CPM) schemes are considered in the context of NBWF. When evaluating the achievable rates, CC-CPM schemes show a 1 to 4 dB maximum margin. In this paper, we investigate on two new sparse graph-based channel coding strategies, trying to reduce this degradation for the achievable rate. Some implementation issues with short codeword lengths are also addressed. To this end, we first consider a serially concatenated CPM scheme where the outer code is an optimized Low-Density Parity-Check code (LDPC) based on some recently introduced methods for this kind of applications. The optimized LDPC scheme exhibits a good bit error rate when binary CPMs are used. This new design comes with lower computational complexity and greater flexibility. Second, in order to avoid the cubersome of iterative detection and decoding, we investigate on recently introduced precoded CPM that can achieve near optimal performance, referred to us as Pragmatic CPM (P-CPM). For this kind of precoded CPM schemes, we will show that associated EXIT chart curves have the flat property. It means that, to achieve good performance, P-CPM can be used with standard capacity approaching codes like LDPC Accumulate Repeat Jagged Accumulate (ARJA) or Turbo Codes without the need for iterative decoding. Bit error rate simulations confirm that P-CPM is a reliable alternative NBWF scheme compatible with versatile modern channel codes. As P-CPM is non-iterative, implementation is made easier. In additive white Gaussian noise, both schemes are found practical.**

I. Introduction

NarrowBand Waveforms (NBWF) deliver critical tactical communications on the battlefield in VHF and UHF military bands ($30 - 512\,\text{MHz}$). Bandwidth is often constraint to $25\,\text{kHz}$. Besides, low latency and robust data rates are important features. These system constraints result in short codeword lengths, which can be challenging for channel coding. The serial concatenation of convolutional codes and CPM (CC-CPM) are commonly considered in this context in conjunction with iterative detection [1], [2]. A recent narrowband waveform is described in [3], [4]. It uses a CC-CPM scheme, for which, at the receiver side, turbo-detection is prescribed [5], [6]. Although CC-CPM schemes offer relatively good performance on additive white Gaussian noise (AWGN) channel, there remain some implementation issues since iterative detection and decoding is usually required to achieve good performance. Moreover, computational complexity increases linearly with iterations.

In this article, we investigate two different serially concatenated coded CPM schemes as alternatives. For each solution, CPM parameters, i.e. modulation index, modulation order and pulse length, are fixed. Thus, waveform properties like bandwidth, constant envelope and spectral masks do not change. Also, user data rate is kept unchanged. As a result, we mainly focus on the design of the outer coding scheme for a fixed CPM modulation scheme. This approach is different than in [7], [8] where CPM parameters are not set.

The first alternative deals with the design for NBWF of a low complexity finite length LDPC code optimized for serial concatenation with a CPM modulation. Recently, a method has been proposed in [9], [10] to design LDPC adapted to CPM for both unstructured and structured LDPC codes. This paper investigates LDPC based solutions adapted to the NBWF CPM. In particular we intend to know if good coding schemes can be designed for small codeword lengths. We will show that careful construction of the parity check matrix should be done in the finite length regime.

The second alternative that is explored is the use of optimized precoded CPM in the context of NBWF, known as Pragmatic-CPM. Pragmatic-CPM has been introduced in [11]. To fully understand the potential of this scheme, we perform an EXIT charts analysis to explain the good performance of the Pragmatic-CPM scheme in general and particularly on NBWF CPM. In particular, it emphasizes the fact that for this kind of precoded CPM, no iterative decoding is required.

This paper is organized as follows. Part II describes the turbo-demodulation scheme with legacy CC-CPM scheme. Part III focuses on the design of LDPC-CPM with small codeword lengths. Finally, in section IV, a non-iterative solution is studied based on P-CPM. For each part, performance of the receiver is assessed in term of bit error rate (BER) on AWGN channel.

TABLE I
CPM AND CODE RATE PARAMETERS FOR NBWF

Mode	L, M, h, Pulse Shape	Code Rate	User bits K (octets)
C0	2, 2, 1/2, REC	1/3	28
C1	2, 2, 1/2, REC	2/3	55
C2	2, 2, 1/4, REC	3/4	86
C3	3, 2, 1/6, REC	4/5	175

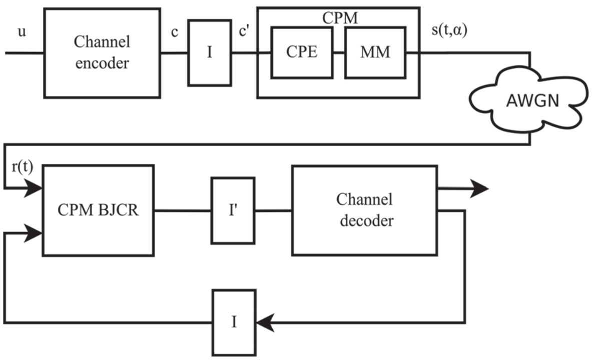

Fig. 1. Channel coding and modulation at transmitter and iterative receiver structure. I and I' blocks represent interleaving and deinterleaving blocks respectively.

II. LEGACY NBWF CC-CPM SCHEME

A. System Description

1) Transmitter: We consider the serial concatenation of different channel codes with a CPM modulation. At the transmitter side, a binary message $\mathbf{u}$ of size K is encoded into a codeword $\mathbf{c}$ of size N, producing a code rate of $R = K/N$. Codeword $\mathbf{c}$ is then interleaved into $\mathbf{c}' = \{c'_n\}_n$ and sent to the CPM modulator. The complex envelope of the CPM signal is given by:

$$s(t,\alpha) = \sqrt{\frac{2E_s}{T}} \exp\left\{ j2\pi h \sum_{n=0}^{N-1} \alpha_n q_{REC}(t-nT) \right\} \quad (1)$$

$$q_{REC}(t) = \begin{cases} 0, \ t \leq 0 \\ \frac{t}{2LT}, \ 0 < t \leq LT \\ 1/2, \ LT < t \end{cases} \quad (2)$$

where E_s is the energy per information symbol, T the symbol interval, $h = k/p$ the modulation index, $\alpha_n = 1 - 2c'_n \in \{-1, 1\}$ the CPM symbol obtained from the binary mapping and $q_{REC}(t)$ the rectangular (REC) phase response. Note, more generic CPM pulse shape could be used. Practically, CPM can be well described using the Rimoldi representation of CPM [12]. This equivalent representation is composed of the serial concatenation of a continuous phase encoder block (CPE) and a memoryless mapper (MM) block. Fig. 1 shows the coding and modulator blocks at both the transmitter and receiver sides. CPM parameters and code rate used throughout this article are listed in Table I.

2) Channel Model and Receiver: At the receiver, the complex envelope of the received signal $r(t)$ is simply modeled as

$$r(t) = s(t,\alpha) + n(t) \quad (3)$$

where $n(t)$ is the complex circular AWGN with double-sided power spectral density $N_0/2$. In this article, phase noise is not included. Carrier frequency offset, time offset and Doppler spread are not considered either. With such channel, we make use of coherent turbo-demodulation employing soft input soft ouput (SISO) detector followed by a SISO channel decoder. Iterations are performed on extrinsic log likelihood ratio between CPM and channel code. Based on the Rimoldi decomposition and the underlying trellis representation, symbol maximum *a posteriori* probability (MAP) decoding can be applied for the CPM SISO demodulator. For the convolutional code, the same is true. Both use the well known BCJR algorithm [13], here the low-complexity max-log map implementation is used. Simulation performance is analyzed using BER versus E_s/N_0. E_s/N_0 is chosen because it does not depend on the bandwidth definition and is consistent with results given in [6], [9].

B. NBWF CPM Achievable Rates

Traditionally, extensive Monte-Carlo simulations on BER or Frame Error Rate (FER) allow designers to assess performance at finite lengths. Asymptotic performance can be assessed for iterative schemes by means of Extrinsic Information Transfer (EXIT) Charts as introduced by [14], see also [15] for a simple introduction. With EXIT Chart, CPM achievable rate can be well estimated with the following property: the area under the demodulator EXIT curve gives the maximum asymptotically achievable R^* for the outer code [16], [15]. Capacity loss, calculated as the area between the code EXIT and demodulator EXIT curves, should be made as small as possible by the designer. Note that exact computation is also possible by estimating the so-called symmetric information rate [11]. In practice, both lead to very close results. Since we consider code design based on an EXIT analysis, we prefer to resort to the first method.

Table II compares asymptotic E_s/N_0 thresholds and simulation results on CC-CPM. CC is instantiated with octal generators $(13, 15, 17)_8$ mother code and suitable puncturing to achieve desired code rate. CC-CPM scheme simulation is performed with 15 iterations. In Table II, difference between R^* and threshold exhibits the capacity loss inherent to the CC-CPM scheme. Simulation results show additional degradation due to the finite length interleaver and the finite codeword lengths. Thresholds found with optimized LDPC based on [9] are systematically better than CC-CPM scheme. LDPC parameters will be described in the next section. These asymptotic margins are the motivation of this paper.

TABLE II
ASYMPTOTIC THRESHOLD E_s/N_0 IN dB FOR CC-CPM AND OPTIMIZED LDPC-CPM. CC-CPM SIMULATION RESULTS ARE ALSO SHOWN.

Mode	C0	C1	C2	C3
R^*	-5.20	-0.56	3.93	10.09
CC-CPM threshold	-3.5	0	4.7	10.8
CC-CPM @BER=10e-4	-1.09	1.91	6.45	12.15
LDPC-CPM threshold ($D_{v,max} = 4$)	-4.47	-0.29	4.40	10.57
LDPC-CPM threshold ($D_{v,max} = 30$)	-5.0	-0.43	3.95	10.24

III. ON LDPC CODE DESIGN FOR NBWF CPM

In this section, instead of a convolutional code, an LDPC code is considered as the outer code. An LDPC code is usually defined using its corresponding binary sparse parity check matrix H of size $M \times N$ with $M = N - K$. A valid codeword belongs to the null space of H. Based on the parity check matrix H, an LDPC code can be alternatively represented by its corresponding Tanner graph [17], consisting in two sets of nodes: the variable nodes associated with the codeword bits (columns of H) and the check node associated with the parity check constraints (rows of H). An edge joins a variable node (VN) n to a check node (CN) m if $H(m,n) = 1$. Edge-perspective degree distribution polynomials $\lambda(x) = \sum_{i=1}^{D_{v,\max}} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{j=2}^{D_{c,\max}} \rho_j x^{j-1}$ are usually used to describe irregular code families. λ_i (resp. ρ_j) is referred to as the proportion of edges in the Tanner graph connected to VN of degree i (resp. to CN of degree j) and $D_{v,\max}$ (reps. $D_{c,\max}$) is the maximum VN (resp. CN) degree.

A. Optimized LDPC Degree Distributions and Protograph Design

Following [9], [18], optimization of LDPC code profiles is performed for CPM parameters detailed in Table II using mutual information (EXIT based) evolution equations. It can be efficiently done using linear programming. For CPM C2, distributions for unstructured LDPC are given in Table III. $D_{v,\max}$ are limited to degree 4 since we are willing to design codes for small codeword lengths. Structured ensembles can be then design based on protograph based LDPC codes [19] which are small bipartite graphs from which larger graphs can be built. In particular, quasi-cyclic LDPC codes can be designed by considering proper lifting of the base matrix associated with the underlying protograph[19]. For the different parameters of the considered CPM waveforms and targeted coding rates, we have search for good base matrices for NBWF CPM, i.e. having good thresholds [9]. The analysis was performed using Protographs EXIT chart (PEXIT) from [20].

B. Finite Lengths LDPC Design

For optimization and asymptotic analysis purposes, partial interleavers between the CPM modulator and the LDPC encoder were introduced. This enables the link with the multi-edge type analysis [21]. Of course, in practice, these partial interleavers can be incorporated within the LDPC interleaver.

TABLE III
OPTIMIZED LDPC DISTRIBUTION FOR CPM C2

C2 LDPC irregular			
λ_1	0.1277	ρ_7	0.15
λ_2	0.6597	ρ_8	0.85
λ_4	0.2127		

It just means that careful design should be done to avoid the presence of small cycles between the CPM and the LDPC during the lifting process. These cycles are new structures that arise when considering the joint graph associated with both the CPM and the LDPC code. This can be done by modifying the PEG-ACE like algorithm to build the matrix H [21].

In final application of NarrowBand Waveforms, codeword lengths are relatively short because data rate is lower compared to satellite applications and because low latency is also a key feature for voice operations. As a result of short codewords, LDPC matrices are harder to construct. Simulations based on LDPC with CPM C2 show a $0.8\,$dB degradation between short codewords of size N=115 octets and codewords of size N=625 octets.

C. LDPC-CPM Complexity Versus CC Complexity

We now analyze the complexity of our scheme. We assume that we use Normalized Min Sum (NMS) decoding. The complexity of one iteration of LDPC with a Normalized Min Sum (NMS) algorithm [22] is evaluated by:

$$C_{iter} = 7 \ intructions \ per \ 1 \ in \ LDPC \ matrix \quad (4)$$

For the CC case, complexity depends on the number of state $N_{state} = 2^m$ where $m = 3$ is code memory. LDPC is more flexible as its complexity is directly function of the number of ones in the matrix which can be adjusted during the LDPC code design with the limitation of $D_{v,max}$. LDPC encoding does not require extra tail bits or tailbiting strategies as with CC. Additionally, scheduling between LDPC and CPM could be optimized. The computational complexity of both solution was evaluated with the number of instructions. CC-CPM has been found more complex than LDPC-CPM scheme. CPM-BCJR block with CPM pulse length $L > 3$ may also be more complex than CC block.

D. Simulation Results

Simulation is performed on AWGN with 10 iterations. An iteration is composed of one CPM demodulator pass followed by one LDPC decoder pass, referred as a $(1, 1)$ scheduling. The LDPC decoder is based on NMS algorithm. Fig. 2 confirms good results of LDPC with NBWF-CPM compared to CPM with convolutional codes. As low as $BER = 10^{-7}$, no error floor was observed. Moreover, poor performance under iterative decoding of standard LDPC codes (such WiMAX, ARJA LDPC codes) have been also observed as reported by [10]. This can be easily explained based on EXIT analysis.

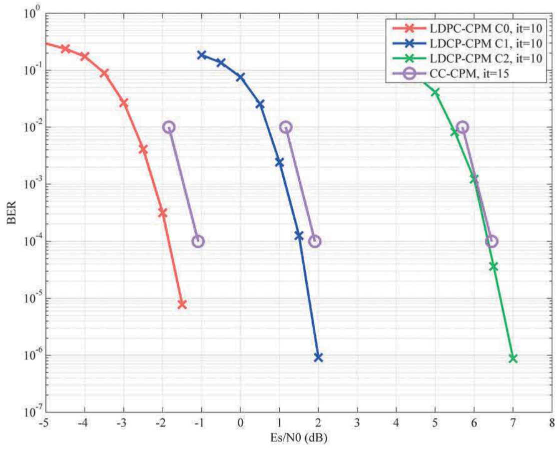

Fig. 2. Monte Carlo simulations of LDPC-CPM versus legacy CC-CPM, for C0, C1 and C2 parameters.

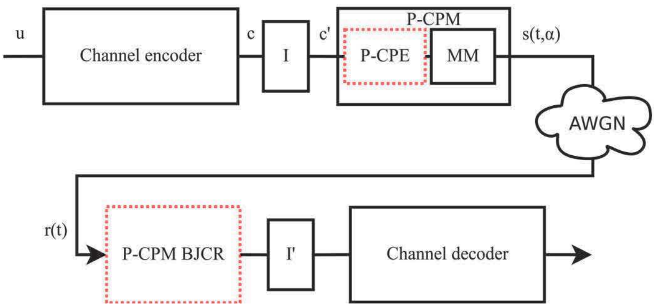

Fig. 3. Tx-Rx block chain for non-iterative detection solution: Pragmatic-CPM.

IV. Pragmatic-CPM Precoding to Avoid Iterative Detection and Decoding

In most cases, due to the inherent memory of CPM, iterative detection and decoding is mandatory to be able to achieve performance close to theoretical limits for serially concatenated coded CPM schemes. This can be easily understood from the EXIT curves of the CPM detector that are not flat in most cases and that are reaching the point $(1, 1)$ for CPM schemes of interest. We can however avoid this situation in a few cases.

A first solution is to take advantage of the implementation of the CPE. For some specific parameters of the CPM waveform [12], the CPE can be written as two equivalent recursive and non-recursive forms (as for convolutional codes, it changes the mapping from input symbol sequences to output symbol sequences). For serially concatenated coded MSK schemes, it was shown in [23] that both CPE representations have transfer functions that are radically different. In particular, the transfer function for the non-recursive MSK scheme can be shown to be flat, showing that a *non-iterative* solution can be implemented at the receiver. Thus, standard capacity approaching coding schemes can be used for the outer channel encoder at the emitter and standard successive soft detection

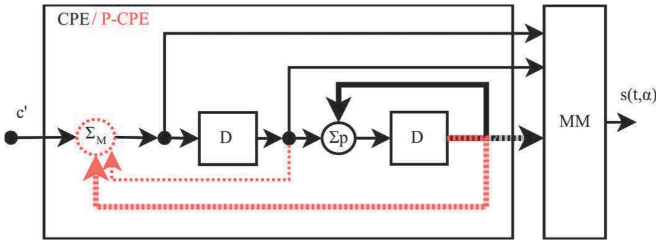

Fig. 4. Continuous Phase Encoder (CPE) for CPM C1 in black solid lines and Optimized pragmatic CPM (P-CPM) in red dotted lines. C1 parameters are M=2, L=2, h=1/2. Block D is a shift register, Σ_M is a modulo M=2 adder and Σ_p is a modulo p adder where p is the denominator of modulation index $h = q/p$.

and decoding is sufficient to achieve near optimal performances. This property has not been further investigated so far. A second solution is the use of a precoding device for the CPM modulator. Efficient solutions can be shown to be incorporated directly within the CPE without or with reasonable increase of complexity. We will show in the following that a recently introduced precoding technique for CPM, referred to us as Pragmatic-CPM [11], can lead to an efficient solution to avoid an iterative detection an decoding at the receiver side. Pragmatic CPM (P-CPM) is a non linear precoding method applied to standard CPM introduced in [11]. The associated scheme is given in Fig. 3. The aim is to design an efficient precoded CPM waveform avoiding the use of iterative detection and decoding. It mainly consists in a modification of the CPE [12] by adding some non linear precoding device. The method is based on the maximization of the so called *pragmatic capacity*, which is in fact the equivalent BICM (Bit-Interleaved Coded-Modulation) capacity as introduced in [24]. It can be easily computed from the outputs of the soft CPM detector. The optimization is performed by selecting the precoding device that allows for this maximization. In the binary case, they derived an optimal structure for the precoded CPM that allows to operate close to the effective capacity of the underlying CPM (i.e. the associated symmetric information rate). For the non binary case, the solution is only proved to be a sub-optimal solution. As an illustration, the modification of the CPM CPE into a CPM Pragmatic-CPE is illustrated in Fig. 4 for the considered waveform. Note that we have checked the spectrum of P-CPM and they are identical to the CPM ones. In the following, we will analyze this scheme based on a EXIT charts analysis to illustrate the good properties of this scheme and we finally derived some conditions on the type of coding schemes that can be used within this context.

A. EXIT Chart for Pragmatic-CPM

We now investigate the performance of the proposed precoded CPM based on EXIT charts. It will offer an alternative analysis to the one proposed in [11]. As shown in Fig. 5 for the particular case of the CPM C2, all binary CPM tested have a flat EXIT curve when precoding is used, unlike legacy CPM. We conjecture that this is true for all binary precoded CPMs that have been shown ”optimal” in [11].

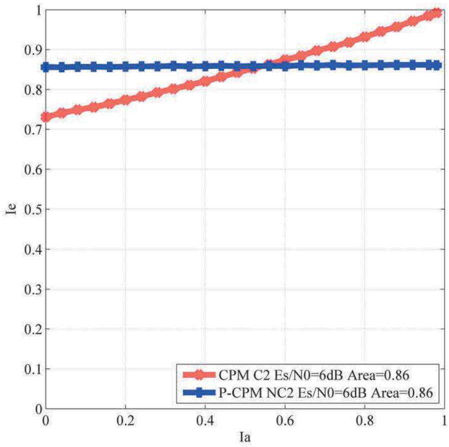

Fig. 5. EXIT Chart for P-CPM C2 and Legacy CPM C2 at Es/N0=6dB with AWGN. Areas under the curve are identical, underlying no loss of capacity associated with the precoding.

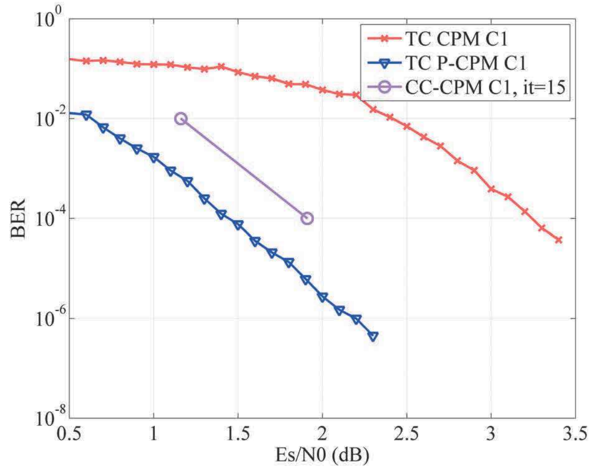

Fig. 6. AWGN Monte-Carlo simulations with C1 parameters using K=55 user octets. It compares TurboCode P-CPM and TurboCode CPM, both without iterations. CC-CPM with 15 external iterations is given as reference.

For the non binary case, this is no longer the case, but this is almost flat and the corresponding EXIT charts are not reaching the point $(1, 1)$. We can also show that the area under the EXIT curves of both precoded and non precoded schemes are almost identical. Following the area theorem (to be more precise, by conjecturing its generalization to general serially concatenated systems) [25], [15], it follows that both schemes can achieve the same achievable rate for the inner code and thus achieve the same spectral efficiency. Thus, we have formally a generalization of the approach considering the recursive and non recursive representations of CPMs that allow for such a representation. Several other properties can be observed.

First, we also observe that the EXIT curve of the precoded scheme does reach the point (1.1). It means that this scheme can no longer be used under iterative decoding with trellis based inner codes such as convolutional codes. It will lead to an unavoidable error floor. As the the EXIT curve is flat, iterative detection and decoding is no longer mandatory and thus capacity approaching codes such as LDPC or turbo-codes will be of interest to achieve near optimal performances. Using convolutional codes will lead to a poor coding gain in that context.

Second, we emphasize the fact that the pragmatic capacity, which is the BICM capacity associated with the soft CPM detector, is given by the point $(0, 1)$ in the EXIT curves. It shows that, indeed, if no iterative decoding is used, the P-CPM will have the best performance for any outer coding scheme. It should also be pointed out that the results are very similar to the results we can have for linear modulations with different mappings. Gray mapped modulations have almost flat EXIT curves for a wide region of signal to noise ratios, thus suggesting that iterative detection and decoding is useless for this signalling scheme. For other modulations schemes, the EXIT is non flat requiring iterative detection and decoding to achieve the same spectral efficiency. Indeed, all mapping have the same area under the EXIT curves. Thus from the aera theorem, we can design efficient coding schemes that have the same achievable rate.

As a conclusion, P-PCM allows for an efficient design of a serially concatenated coded CPM scheme without iterative decoding. Any efficient scheme such as turbo-codes or LDPC codes can be used as outer codes. This greatly simplifies the implementation issues. It permits to use co-processor already available on many system-on-a-chip like LTE turbo code, WIMAX or ARJA LDPC codes.

Note that ”true” capacity approaching coding schemes can be used by properly designing the outer coding schemes to match the true statistics at the outputs of the soft demodulator. This can be efficiently done by using LDPC codes or other codes on graphs.

B. Simulation Results

We consider the LTE Turbo Code as outer code concatenated wih CPM or with P-CPM. Simulations are performed on an AWGN channel with CPM C1 parameters or CPM C3 parameters, without iterative detection and decoding. On results reported on Fig. 6, coding rate is set to 2/3 whereas on Fig. 7, coding rate is set to 4/5. It is shown that the Turbo Code with P-CPM is, with no surprise, from far better than Turbo Code concatenated with legacy CPM. When iterative detection and decoding is considered for the legacy CC-CPM scheme, the results are still slightly in favor of the Turbo Code P-CPM scheme.

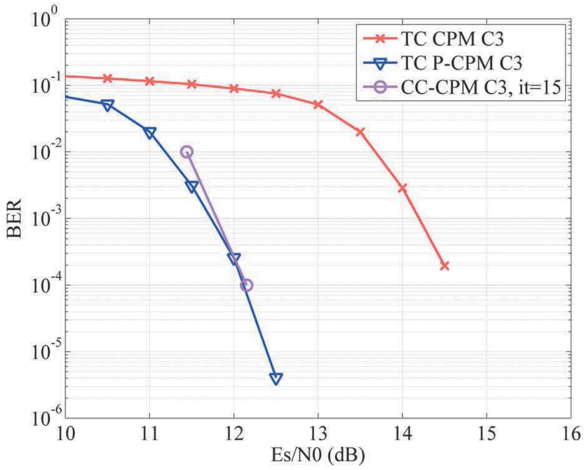

Fig. 7. AWGN Monte-Carlo simulations with C3 parameters using K=175 user octets.

V. Conclusion

This paper proposed channel coding alternatives to serially concatenated convolutional codes and CPM in the context of short codeword lengths. First, it explored a method to design an LDPC code finely matched to CPM specificity. We found out that the underlying complexity was reduced with LDPC introduction and the bit error rate performance is improved on AWGN channel. The other proposition was a CPM precoding, called Pragramtic-CPM, witch flattens the CPM EXIT chart. Thanks to EXIT Chart analysis, we showed Pragmatic-CPM can be used with modern channel codes like turbo-codes or ARJA LDPC already available as COTS. This assumption was confirmed on AWGN simulations with binary CPM.

Further work is required to assess performance with more realistic channel models. It could include the introduction of carrier frequency offsets, phase noise, multipath channels and jamming.

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